FLUIDS AT REST.

A fluid is a substance which can flow. Therefore, liquids and gases which have the ability to flow can be called fluids, in general.

Fluids can be categorized into two, namely; Hydro-statics and Hydro dynamics.

HYDRO-STATICS.

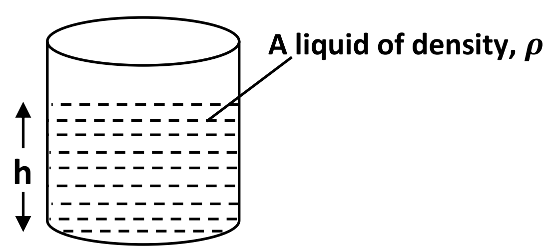
Hydro-statics deals with fluids at rest. One of the most important concepts in connection with fluids is pressure.

PRESSURE.

The pressure acting on a surface is the force per unit area acting normally on the surface.

PRESSURE IN FLUIDS.

Consider a fluid, say a liquid of density ρ, filled in a container of cross section area A, to a depth h, as shown below;



pressure, P = $\frac{Force}{Area}$

P = $\frac{mg}{A}$

where m is the mass of the liquid

m = volume x density

m = Ah x ρ

Hence, pressure, P = $\frac{Ah\ x\ \rho\ x\ g}{A}$

P = hρg. .... eqn (i)

The equation above confirms that pressure in fluids depends on the density and the depth but also, that pressure is independent of the cross section area of the container in which the fluid is filled.

NOTE:

(i) The S I unit of pressure is Pascal (Pa). However, 1 Pa = 1 N m^{-2}.

(ii) The pressure at a point in a fluid is transmitted equally in all directions, hence pressure is a scalar quantity.

(iii) Pressure in liquids increases with depth.

DENSITY (ρ)

Density of a substance is defined as the mass per unit volume of a substance, ie

Density = $\frac{mass}{volume}$

The S.I unit of density is kg m^{-3}. However, other units of density are g cm^{-3}.

It should be noted that 1 g cm^{-3} = 1000 kg m^{-3}.

RELATIVE DENSITY (R.D)

Relative density can be defined as the ratio of the density of a substance to the density of an equal volume of water.

Relative density = $\frac{density\ of\ a\ substance}{density\ of\ water}$

Relative density has no units and therefore it is dimensionless.

It is supreme to recall that in addition to the formula above, relative density can also be obtained from;

Relative density of a solid = $\frac{weight\ of\ the\ object}{weight\ of\ an\ equal\ volume\ of\ water}$

$$\text{R.D} = \frac{Weight\ in\ air}{Weight\ in\ air - Weight\ in\ water}$$

Relative density of a liquid = $\frac{Upthrust\ in\ a\ liquid}{Upthrust\ in\ water}$

$$\text{R.D} = \frac{Weight\ in\ air - Weight\ in\ a\ liquid}{Weight\ in\ air - Weight\ in\ water}$$

An experiment to determine the density of an irregular object which floats on water.

- An irregular object is suspended from a spring balance, using a piece of thread, and its weight in air, W_a is read and recorded.
- A sinker such as a stone is attached to the irregular object and their total weight W_2 when completely immersed in water is determined from the spring balance.
- The irregular object is detached from the sinker, and the weight, W_3 of the sinker alone, when completely immersed in water is noted.
- The weight, W_w of the irregular object in water can be calculated from; $W_w = W_2 - W_3$
- The relative density of the irregular object is obtained using

$$\text{R.D} = \frac{W_a}{W_a - W_w}$$

The density of the irregular object can also be determined from

density of the object = R.D x density of water.

EXAMPLES: (Skip some spaces for the solutions that follow)

1. A spherical stone has a mass of 1.546 kg and its radius is 20 cm. Find the relative density of the stone in kg m^{-3}. $(Ans \quad R.D = 0.04615)$
2. An object suspended from a spring balance is found to have a weight of 4.92N in air and 3.87N in water. Calculate the density of the material from which the object is made if the density of water is 1000 kg m^{-3}.

$$(Ans \quad \rho = 4686\ kgm^{-3})$$

3. A solid weighs 20.0g in air, 15.0g in water and 16.0g in a liquid R. Calculate the relative density of liquid R. $(Ans\ R.D = 0.8\ )$

ARCHIMEDES' PRINCIPLE.

It states that, when a body is totally or partially immersed in a fluid, it experiences an upthrust equal to the weight of the displaced fluid.

UPTHRUST.

Is the upward force exerted on a body immersed in a fluid.

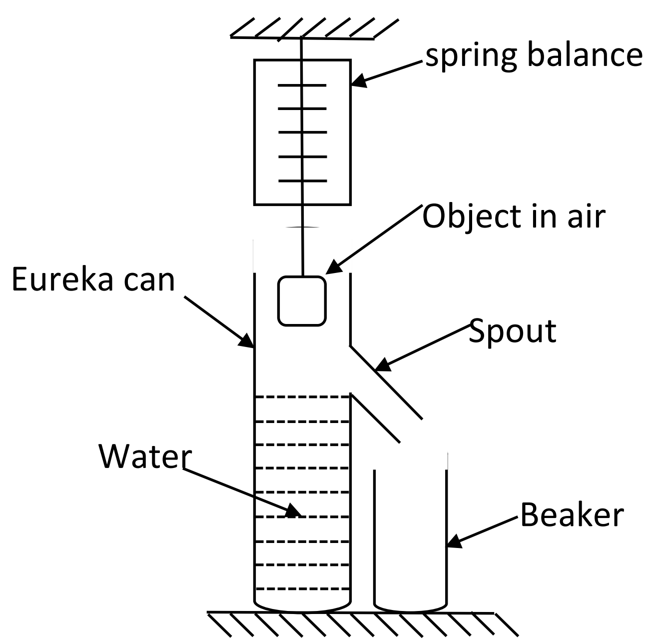
According to Archimedes,

Upthrust = weight of the displaced fluid

BUOYANCY.

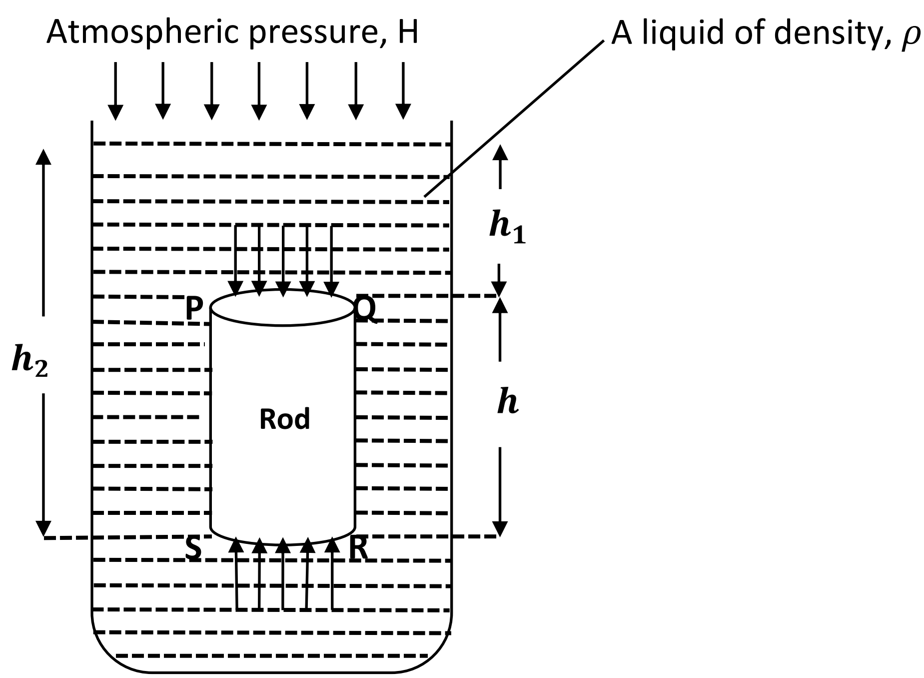
Is the tendency of an object to move upwards (rise) when immersed in a fluid. For this reason, upthrust can also be called a buoyant force while such like an object which is immersed into a fluid, can be called a buoy.

AN EXPERIMENT TO VERIFY ARCHIMEDES' PRINCIPLE USING A SPRING BALANCE.



- An object is suspended from a spring balance, by means of a thread, and its weight, W_a in air is read and recorded.
- The Eureka can is filled with water up to the spout level, with a beaker placed beneath the spout.
- The object is gently lowered into water, and its weight, W_w when completely immersed in water is read and recorded.
- The upthrust, U is calculated from $U = W_a - W_w$
- The displaced water from the can to the beaker is weighed and its Weight, W is obtained.
- It is observed that $W = W_a - W_w$ which verifies Archimedes' principle.

TO SHOW THAT THE WEIGHT OF THE DISPLACED FLUID BY AN OBJECT IS EQUAL TO THE UPTHRUST ON THE OBJECT.



Consider a cylindrical rod PQRS of cross section area, A and height, h totally immersed in a liquid of density, ρ as shown above.

The total pressure at the base of the rod = H + ρgh_2

Hence, force at the base, F_2 = (H + ρgh_2)A upwards

Likewise, force at the top, F_1 = (H + ρgh_1)A downwards

Since h_2 is greater than h_1 then F_2 is greater than F_1.

Resultant Upward force, U = $F_2 – F_1$

$$U = (H + \rho gh_2)A – (H + \rho gh_1)A$$

$$U = \rho g(h_2 – h_1)A \quad \text{where } h_2 – h_1 = h$$

$$U = \rho ghA \quad \text{..} \quad (i)$$

However, volume of the rod = A x h

Weight of the liquid displaced = volume x density of liquid x g

= A x h x ρ x g

= Ahρg .... (ii)

Since equations (i) and (ii) are equal, then upthrust is equal to the weight of the displaced liquid.

EXAMPLES.

1. A string supports a solid iron object of mass 0.18 kg totally immersed in a liquid of density 800 kg m^{-3}. Calculate the tension in the string given that the density of iron is 8000 kg m^{-3}. $(Ans \quad T = 1.59N)$

2. An alloy contains two metals A and B. It has a volume of 5.0 x 10^{-4} m^3 and density of 5.6 x 10^3 kg m^{-3}. The densities of A and B are 8.0 x 10^3 kg m^{-3} and 4.0 x 10^3 kg m^{-3}. Calculate the mass of A and B.
$(Ans \quad M_A = 1.6kg,\ M_B = 1.2kg)$

3. An alloy contains two metals X and Y of densities 3.0 x 10^3 kg m^{-3} and 5.0 x 10^3 kg m^{-3} respectively. Calculate the density of the alloy given that;
   (i) the volume of metal X is twice that of metal Y.
   $(Ans\ \rho = 3.67\ x\ 10^3 kgm^{-3})$

(ii) the mass of metal X is twice that of metal Y.

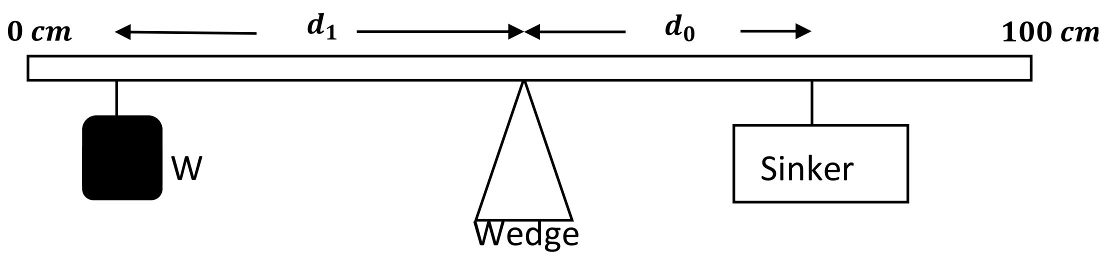
$(Ans\ \ \rho = 3.46\ x\ 10^3 kgm^{-3})$

4. A string supports a metal block of mass 4 kg which is completely immersed in a liquid of density 6.2 x 10^2 kg m^{-3}. If the density of the block is 8.5 x 10^3 kg m^{-3}, calculate the tension in the string attached to the block.

$(Ans\ \ T = 36.4N)$

AN EXPERIMENT TO DETERMINE THE RELATIVE DENSITY OF A LIQUID, USING ARCHIMEDES' PRINCIPLE AND THE PRINCIPLE OF MOMENTS.

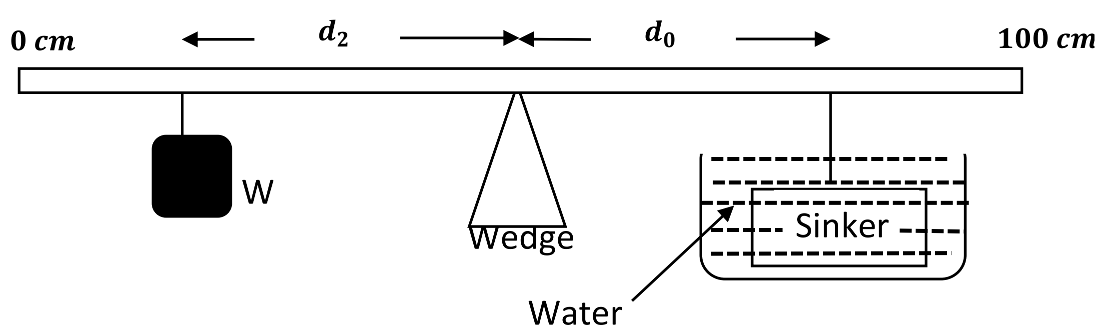
- A weight, W and a sinker are suspended from a metre rule with the wedge in between, as shown below.



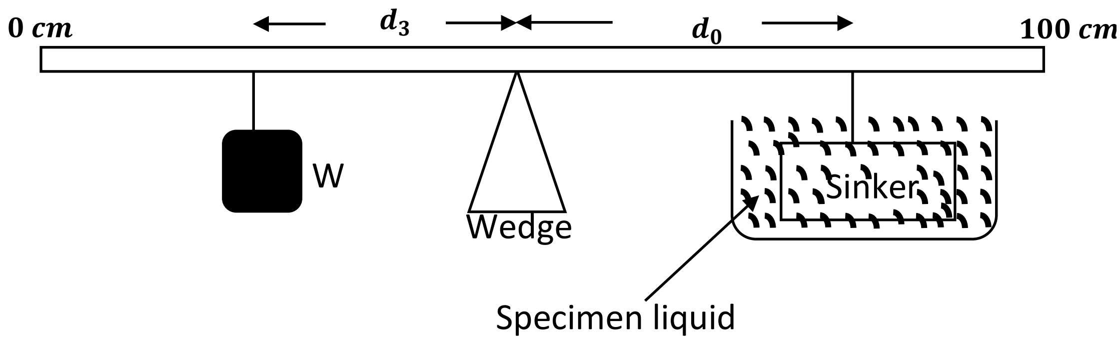
- The position of the weight W is adjusted until the metre rule balances horizontally. The distances d_1 and d_0 are measured and recorded.
- Using the principle of moments, the weight of the sinker, W_a is obtained from

$$W_a = W \times \frac{d_1}{d_0} \text{ .... (i)}$$

- The sinker is then completely immersed in a beaker of water, while keeping d_0 constant, as shown below.



- The position of the weight W is adjusted until balance is restored and the new distance d_2 is measured and noted.
- Using the principle of moments, the weight of the sinker, W_w in water is obtained from $W_w = W \times \frac{d_2}{d_0}$ .... (ii)
- The sinker is removed from water and placed in a beaker containing a specimen liquid, while keeping d_0 constant as shown below.



- The position of the weight W is adjusted until balance is restored and the distance d_3 is measured and noted.
- The weight of the sinker, W_L in the liquid is obtained from

$$W_L = W \times \frac{d_3}{d_0} \text{ .... (iii)}$$

The relative density of the liquid can be determined using

$$R.D = \frac{W_a - W_L}{W_a - W_w}$$

Substituting equations (i), (ii) and (iii)

$$R.D = \frac{d_1 - d_3}{d_1 - d_2}$$

THE LAW OF FLOATATION.

It states that, a floating body displaces its own weight of the fluid in which it floats. That is to say;

"Weight of the floating body = weight of the displaced fluid."

However, since weight is proportional to mass,

Mass of the floating body = mass of the displaced fluid.

N.B

density of a floating body = fraction submerged x density of the fluid

EXAMPLES.

1. An object floats in a liquid of density 1200 kg m^{-3} with $\frac{1}{4}$ of its volume above the liquid surface. Calculate the density of the floating object.

$$(Ans \ \ \rho = 900kgm^{-3})$$

2. A block of wood floats in water of density 1000 kg m^{-3} with $\frac{2}{3}$ of its volume submerged. However, in oil, it has $\frac{9}{10}$ of its volume submerged. Find the densities of wood and oil.

$$(Ans \ \ \rho_{wood} = 666.7kgm^{-3} \ , \ \rho_{oil} = 740.7kgm^{-3})$$

3. A hydrometer floats in water with 72% of its volume submerged. The hydrometer floats in another liquid with 80% of its volume submerged. Find the relative density of the liquid if the density of water is 1000 kg m^{-3}.

$$(Ans \quad R.D = 0.90\ )$$

4. An object with a volume of 32 cm^3 floats in water with exactly half of the object below the liquid surface. If the density of water is 1000 kg m^3, calculate the mass of the floating object. $(Ans \quad m = 0.016kg)$

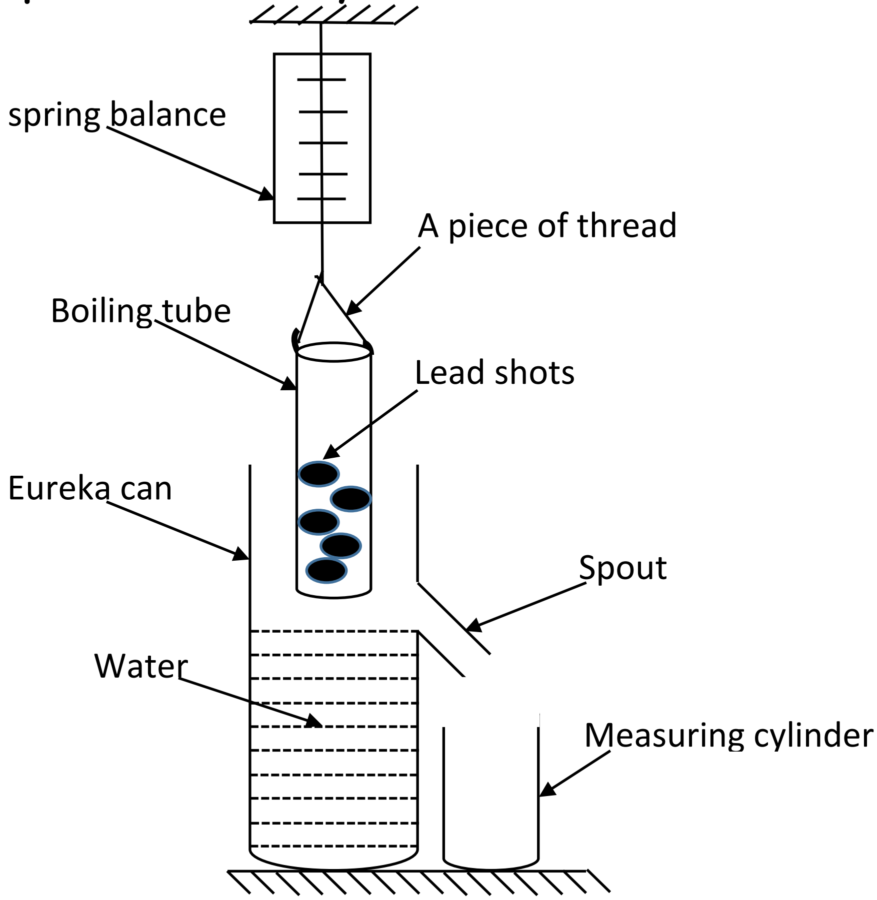
5. A solid of volume 1.0 x 10^{-4} m^3 floats in water of density 1000 kg m^{-3} with $\frac{3}{5}$ of its volume submerged.

   (i) Find the mass of the solid.

   (ii) If the solid floats in another liquid with $\frac{4}{5}$ of its volume submerged, what is the density of the liquid.

$$(Ans \quad m = 0.060kg \ , \ \ \rho = 750kgm^{-3})$$

An experiment to verify the law of floatation.



- Water is filled in a Eureka can up to the spout level, with a measuring cylinder below the spout, as shown above.
- A glass tube containing lead shots **of known weight** is gently lowered into the can such that it floats vertically.
- The weight of the displaced water in the measuring cylinder is measured and recorded.
- Observations show that the weight of the displaced water is equal to the weight of the floating tube, which verifies the law of floatation.

MORE EXAMPLES.

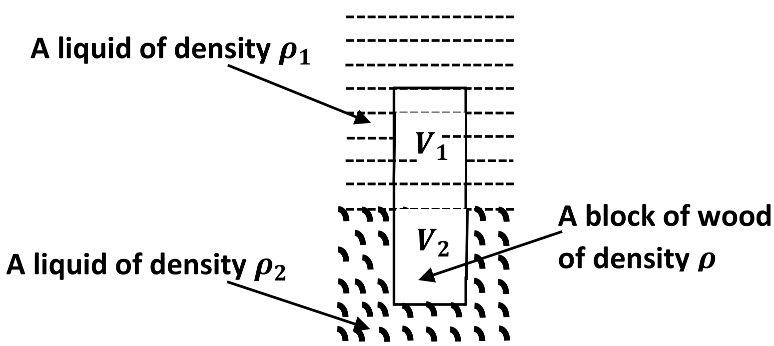
1. A solid weighs 237.5 g in air and 12.5 g when totally immersed in a fluid of density 0.9 g cm^{-3}. Calculate the ;

(i) Density of the solid

(ii) Density of the liquid in which the solid would float with $\frac{1}{5}$ of its volume exposed above the liquid surface.

$$(Ans\ \ \rho_{solid} = 950kgm^{-3}, \rho_{liquid} = 1187.5kgm^{-3})$$

2.



A block of wood of density ρ floats in an interface between immiscible liquids of densities ρ_1 and ρ_2 as shown above. Show that the ratio of the volume **V_1 : V_2** of the block in the two liquids is given by;

$$\frac{V_1}{V_2} = \frac{\rho_2 - \rho}{\rho - \rho_1}$$

3. A piece of metal of mass 2.60×10^{-3} kg and density 8.4×10^{3} kg m^{-3} is attached to a block of wax of mass 1.0×10^{-2} kg and density 9.2×10^{2} kg m^{-3}. When the system is placed in a liquid, it floats with wax just submerged. Find the density of the liquid. $(Ans\ \ \rho_{liquid} = 1127kgm^{-3})$

TRIAL QUESTIONS.

1. A string supports a solid copper block of mass 1kg and density 9×10^{3} kg m^{-3} which is completely immersed in water of density 1×10^{3} kg m^{-3}. Calculate the tension in the string. $(Ans\ \ T = 9.0N)$

2. A hydrometer floats in water with 68% of its volume submerged. The

hydrometer floats in another liquid P, with 78% of its volume submerged. Calculate the relative density of the liquid P. $(Ans\ \ R.D = 0.87)$

3. The mass of a specimen of an alloy of silver and gold whose densities are 10.5 g cm^{-3} and 18.9 g cm^{-3} respectively is 35.2g in air, and 33.13g in water. Find by composition, the mass of the alloy assuming that there has been no Volume change in the process of producing the alloy.
(take density of water = 1 g cm^{-3})

$$\left(Ans\ \ m_{silver} = 4.9035g\ ,\ m_{gold} = 30.2963g\right)$$

4. A tube of uniform cross-section area 4.0 x 10^{-3} m^{2} and mass of 0.2 kg is separately floated vertically in water of density 1.0 x 10^{3} kg m^{-3} and in oil of density 8.0 x 10^{2} kg m^{-3}. Calculate the difference in the length immersed.

$(Ans\quad h = 0.0125m)$

5. A block of wood floats in an interface between water and oil with 0.25 of its volume submerged in water. If the density of the wood is 7.3 x 10^{2} kg m^{-3}, calculate the density of oil. $(Ans\ \ \rho_{oil} = 640kgm^{-3})$

6. A hydrometer consists of a spherical bulb and a cylindrical stem which has a a cross section area of $0.6\ cm^2$. The total volume of the bulb and the stem is $14.3\ cm^3$. When immersed in water, the hydrometer floats with $7.6cm$ of the stem above the water surface. When in alcohol, it floats with $2.0cm$ of the stem above the alcohol surface. Given that the density of water is $1\ gcm^{-3}, calculate\ the\ density\ of\ alcohol$.

$(Ans\quad \rho = 0.744gcm^{-3})$

END: GOOD LUCK.

STAY HOME STAY SAFE.